**Investments Problem Sheet 3 Lent Term 2024**

**True-False**

1. The best way of estimating the future beta of a share is to use the historic beta, computed by regressing the share return on the market return.

**Questionable. If the business of the firm has not changed and nor has its financial leverage, then historic beta is a good starting point.**

1. If the stock market index goes down, high beta shares are also likely to go down, but not by as much as the market index.

**False. If the CAPM is true, the expected return on a share, *r*, given that the market return is *rm*, is *rf + *(*rm-rf*). In general, if *rm* is less than *rf*,the higher the beta, the lower the expected return.**

1. The evidence that high beta shares have a higher expected return than low beta shares, after controlling for other factors, is weak.

**True. Fama and French (1992) for example show that after controlling for factors such as size and book-to-market ratio, there is no significant difference between high and low beta shares.**

**Questions**

1. Assets A and B are predicted to have the following returns in the five possible states of the world next period:



What is the expected return and standard deviation of returns on A and B? What is the correlation between the returns on A and the returns on B? Given that the only two assets you can invest in are A and B, is there any reason why anyone should want to buy some of B?

**The calculations involves the standard formulae, and are contained in the following embedded spreadsheet (right click on the soft version, and use “worksheet object/open):**

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**The means are obtained by multiplying each of the returns by the probability (using the sumproduct function); having obtained the means, the squared deviation from the means are computed, and these are multiplied by the probabilities to give the variance, and then square rooted to give the standard deviation. The correlation is obtained by dividing the covariance by the product of the two standard deviations.**

**Although B has lower mean and higher standard deviation than A, it might well be attractive to someone who values money in states 2 and 5 (whatever they may be) when B will beat A.**

2. Mr Entrepreneur’s entire wealth of £50m consists of shares in the company he founded. He is not happy holding such an undiversified portfolio, but is unwilling to sell any of his shares because he does not want to lose control of the company. A friend suggests that he could diversify by borrowing some money from the bank against the security of his shares and invest it in the stock market. In this way he would have a much more diversified portfolio. The bank is ready to lend him up to £25m. Can he reduce his total risk by doing as his friend suggests? Explain carefully.

**Dollar volatility of existing shares = £50m\*σexisting shares**

**Dollar volatility of new shares = £25m\*σnew shares**

**If dollar volatility of existing shares is a and of new shares is b, then variance of new portfolio is a2+2rab+b2, where r is the correlation. This is only less than the original a2 if r < -b/2a. The existing shares must have negative correlation with the market (negative beta) to get any benefit at all.**

**Problems**

1. An investor can invest in two risky assets 1 and 2, and the risk-free asset. The expected returns on the risky assets are 1 = 0.15, 2 = 0.2, and their variance-covariance matrix is

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(so each share has a volatility of √0.3, or about 55%, and their correlation is 2/3). The risk-free rate *RF* is 5%.

(a) What is the expected return and standard deviation of returns for a portfolio with:

(i) 50% invested in each of the two risky assets?

(ii) 25% invested in asset 1 and 75% invested in asset 2?



(b) Compute the *Sharpe ratio* for the four portfolios (100,0;25,75;50,50;0,100) where the Sharpe ratio is defined as



and sketch on a graph.

(c) Pretend that asset 2 is the market portfolio. Compute its correlation with the other portfolios, and their betas on it. Verify whether or not the Capital Asset Pricing Model holds if asset 2 is indeed the market portfolio.

(b) Compute the *Sharpe ratio* for the four portfolios (100,0;25,75;50,50;0,100) where the Sharpe ratio is defined as and sketch on a graph.

**The answers to parts (a) and (b) are in the embedded spreadsheet. The portfolio that has 0 in share 1 and 100% in share 2 has the highest Sharpe Ratio.**



(c) Pretend that asset 2 is the market portfolio. Compute its correlation with the other portfolios, and their betas on it. Verify whether or not the Capital Asset Pricing Model holds if asset 2 is indeed the market portfolio.

**The beta of share 1 on share 2 is cov(1,2)/var(2) = 2/3. The risk free rate is 5%, and the expected return on the “market” (share 2) is 20%, so the expected return on share 1 should be 5% + 2/3(20%-5%) = 15% - just as the “CAPM” would predict. In general, if you take M as the portfolio with maximum Sharpe ratio, then the “CAPM” relationship will hold for any asset with M as the market portfolio.**

2. The following annual excess rates of return were obtained for nine individual stocks and a market index.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | Market Excess Returns(%) | A | B | C | D | E | F | G | H | I |
| 1 | 29.65 | 33.88 | -25.20 | 36.48 | 42.89 | -39.89 | 39.67 | 74.57 | 40.22 | 90.19 |
| 2 | -11.91 | -49.87 | 24.70 | -25.11 | -54.39 | 44.92 | -54.33 | -79.76 | -71.58 | -26.64 |
| 3 | 14.73 | 65.14 | -25.04 | 18.91 | -39.86 | -3.91 | -5.69 | 26.73 | 14.49 | 18.14 |
| 4 | 27.68 | 14.46 | -38.64 | -23.31 | -0.72 | -3.21 | 92.39 | -3.82 | 13.74 | 0.09 |
| 5 | 5.18 | 15.67 | 61.93 | 63.95 | -32.82 | 44.26 | -42.96 | 101.67 | 24.24 | 8.98 |
| 6 | 25.97 | -32.17 | 44.94 | -19.56 | 69.42 | 90.43 | 76.72 | 1.72 | 77.22 | 72.38 |
| 7 | 10.64 | -31.55 | -74.65 | 50.18 | 74.52 | 15.38 | 21.95 | -43.95 | -13.40 | 28.95 |
| 8 | 1.02 | -23.79 | 47.02 | -42.28 | 28.61 | -17.64 | 28.83 | 98.01 | 28.12 | 39.41 |
| 9 | 18.82 | -4.59 | 28.69 | -0.54 | 2.32 | 42.36 | 18.93 | -2.45 | 37.65 | 94.67 |
| 10 | 23.92 | -8.03 | 48.61 | 23.65 | 26.26 | -3.65 | 23.31 | 15.36 | 80.59 | 52.51 |
| 11 | -41.61 | 78.22 | -85.02 | -0.79 | -68.70 | -85.71 | -45.64 | 2.27 | -72.47 | -80.26 |
| 12 | -6.64 | 4.75 | 42.95 | -48.60 | 26.27 | 13.24 | -34.34 | -54.47 | -1.50 | -24.46 |

1. Perform the first-stage regressions and tabulate the summary statistics

**Using the regression feature of Excel with the data presented in the text, the first-stage time-series (Security Characteristic Line) estimation results are:**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Stock | A | B | C | D | E | F | G | H | I |
| *R*-square | 0.06 | 0.06 | 0.06 | 0.37 | 0.17 | 0.59 | 0.06 | 0.67 | 0.70 |
| Observations | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| Alpha | 9.00 | -0.63 | -0.64 | -5.05 | 0.73 | -4.53 | 5.94 | -2.41 | 5.92 |
| Beta | -0.47 | 0.59 | 0.42 | 1.38 | 0.90 | 1.78 | 0.66 | 1.91 | 2.08 |
| *t*-Alpha | 0.73 | -0.04 | -0.06 | -0.41 | 0.05 | -0.45 | 0.33 | -0.27 | 0.64 |
| *t*-Beta | -0.81 | 0.78 | 0.78 | 2.42 | 1.42 | 3.83 | 0.78 | 4.51 | 4.81 |

1. Specify the hypothesis for a test of the second-stage cross-sectional regression for the Security Market Line.

**Answer: The hypotheses for the second-stage regression for the Security Market Line are:**

* **The intercept is zero.**
* **The slope is equal to the average return on the index portfolio.**

1. Perform the second-stage cross-sectional regression (Security Market Line regression) by regressing the average excess return of each portfolio on its beta.

**Answer**: The second-stage data from the first-stage time-series (SCL) estimates are:

|  |  |
| --- | --- |
| Average Excess Return | Beta |
| A 5.18 | -0.47 |
| B 4.19 | 0.59 |
| C 2.75 | 0.42 |
| D 6.15 | 1.38 |
| E 8.05 | 0.90 |
| F 9.90 | 1.78 |
| G 11.32 | 0.66 |
| H 13.11 | 1.91 |
| I 22.83 | 2.08 |

The second-stage cross-sectional regression yields:

Regression Statistics

Multiple R 0.7074

R-square 0.5004

Adjusted R-square 0.4291

Standard error 4.62 Observations 9

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Coefficients | Standard Error | *t* Statistic for β=0 | *t* Statistic for β=8.12 |
| Intercept | 3.92 | 2.54 | 1.54 |  |
| Slope | 5.21 | 1.97 | 2.65 | -1.48 |

1. Summarize your test results and compare them to the results reported in the text.

**Answer: As we saw in the chapter, the intercept is too high (3.92% per year instead of 0) and the slope is too flat (5.21% instead of a predicted value equal to the sample-average risk premium: rM - rf = 8.12%). The intercept is not significantly greater than zero (the t-statistic is less than 1.96) and the slope is not significantly different from its theoretical value (the t-statistic for this hypothesis is -1.48). This lack of statistical significance is probably due to the small size of the sample.**

1. Group the nine stocks into three portfolios, maximizing the dispersion of the betas of the three resultant portfolios. Repeat the test and explain any changes in the results.

**Answer:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Year | | ABC | | DEG | | FHI |
| 1 | | 15.05 | | 25.86 | | 56.69 |
| 2 | -16.76 | | -29.74 | | -50.85 | |
| 3 | 19.67 | | -5.68 | | 8.98 | |
| 4 | -15.83 | | -2.58 | | 35.41 | |
| 5 | 47.18 | | 37.70 | | -3.25 | |
| 6 | -2.26 | | 53.86 | | 75.44 | |
| 7 | -18.67 | | 15.32 | | 12.50 | |
| 8 | -6.35 | | 36.33 | | 32.12 | |
| 9 | 7.85 | | 14.08 | | 50.42 | |
| 10 | 21.41 | | 12.66 | | 52.14 | |
| 11 | -2.53 | | -50.71 | | -66.12 | |
| 12 | -0.30 | | -4.99 | | -20.10 | |
| Average | 4.04 | | 8.51 | | 15.28 | |
| Std. Dev. | 19.30 | | 29.47 | | 43.96 | |

The first-stage time-series (SCL) estimates are:

|  |  |  |  |
| --- | --- | --- | --- |
| *R*-square | ABC | DEG | FHI |
| 0.04 | 0.48 | 0.82 |
| Observations | 12 | 12 | 12 |
| Alpha | 2.58 | 0.54 | -0.34 |
| Beta | **0.18** | **0.98** | **1.92** |
| *t*-Alpha | 0.42 | 0.08 | -0.06 |
| *t*-Beta | 0.62 | 3.02 | 6.83 |

Grouping into portfolios has improved the SCL estimates as is evident from the higher R-square for Portfolio DEG and Portfolio FHI. This means that the beta (slope) is measured with greater precision, reducing the error-in-measurement problem at the expense of leaving fewer observations for the second pass.

The inputs for the second-stage cross-sectional regression are:

|  |  |  |
| --- | --- | --- |
|  | Average Excess Return | Beta |
| ABC | 4.04 | 0.18 |
| DEH | 8.51 | 0.98 |
| FGI | 15.28 | 1.92 |
| M | 8.12 |  |

The second-stage cross-sectional estimates are:

Regression Statistics

Multiple *R* 0.9975

*R*-square 0.9949

Adjusted *R*-square 0.9899

Standard error 0.5693

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Observations | 3 |  |  |  |
|  | *Coefficients* | *Standard Error* | *t Statistic for β =0* | *t Statistic for β =8.12* |
| Intercept | 2.62 | 0.58 | 4.55 |  |
| Slope | 6.47 | 0.46 | 14.03 | -3.58 |

Despite the decrease in the intercept and the increase in slope, the intercept is now significantly positive, and the slope is significantly less than the hypothesized value by more than three times the standard error.